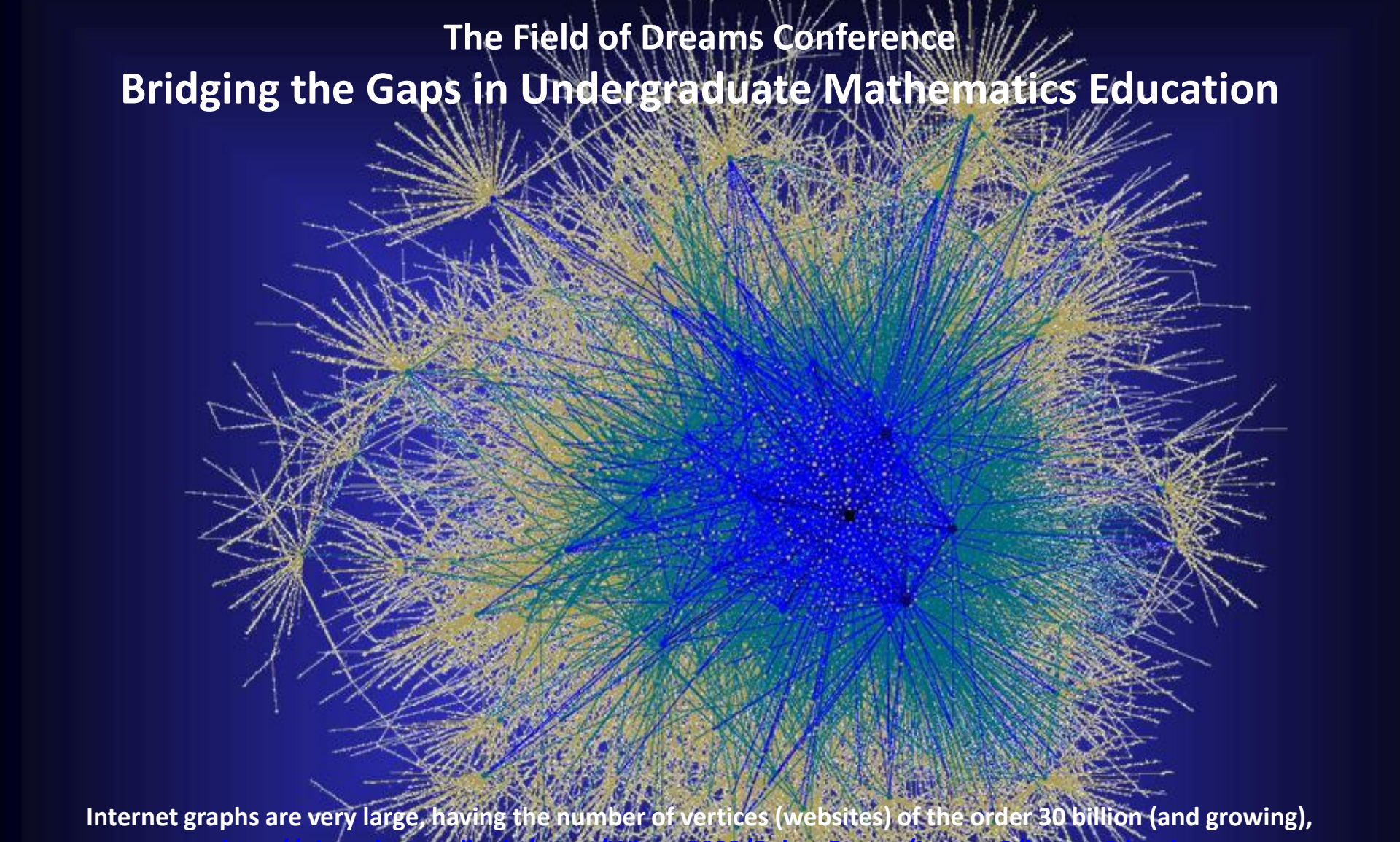
The Field of Dreams Conference
Bridging the Gaps in Undergraduate Mathematics Education



Internet graphs are very large, having the number of vertices (websites) of the order 30 billion (and growing),
http://pi.math.cornell.edu/~mec/Winter2009/RalucaRemus/Lecture2/lecture2.html

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Introduction

This talk explores some of the strategies being used by mathematics departments to complement classroom lectures and text books in a manner that empowers and inspires students to be independent learners. For all students, learning should be an enjoyable experience of acquiring knowledge, building confidence, and developing the skills necessary to become effective problem solvers.

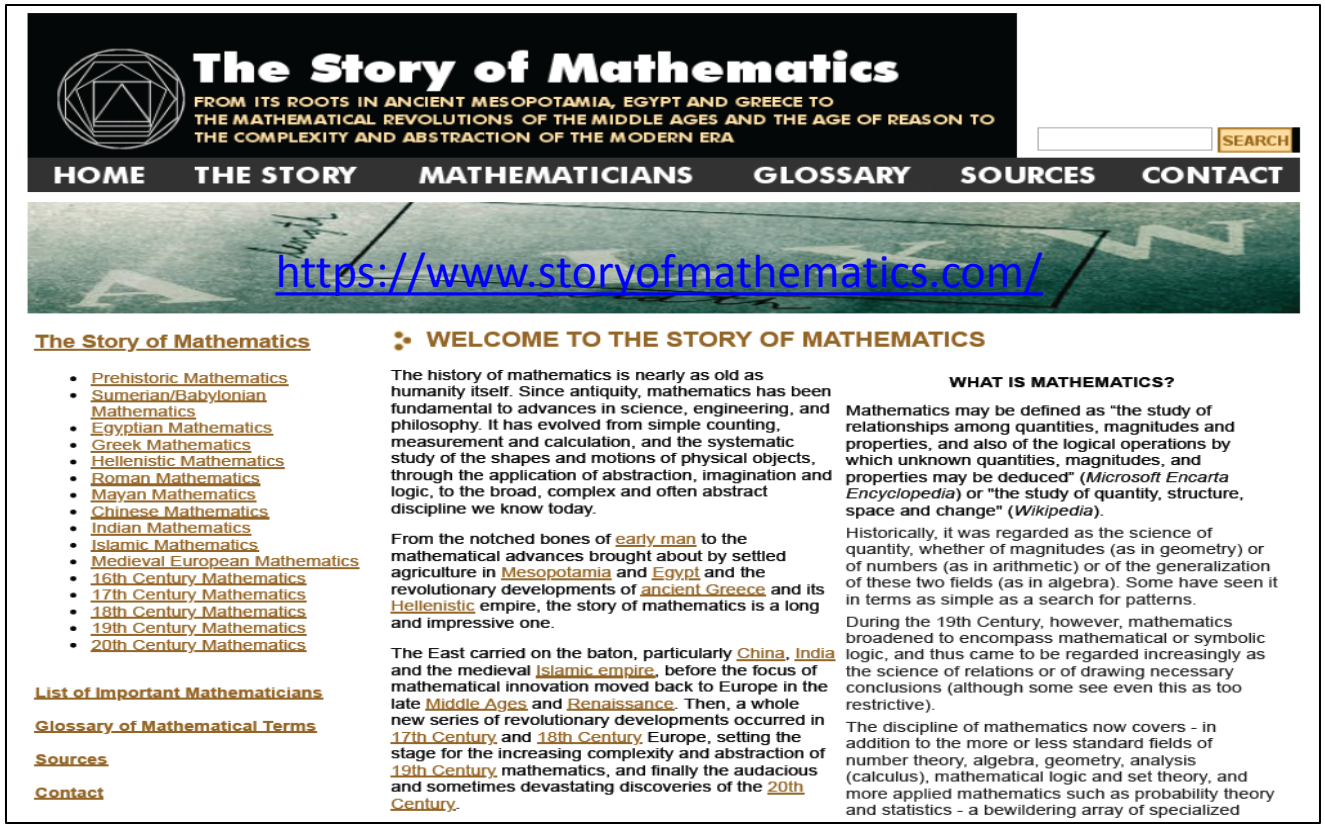
Today's classrooms are very diverse. Many of the students fall into the following categories;

- The ideal students: those who study faithfully according to their needs; will read the text, work many of the exercises in the text, complete homework assignments,
- attend every class and reach out for help if needed. They are committed to excellence.
- Well prepared students: those with the necessary background to succeed in a subject;
- Nontraditional students: those who defer college in order to work or for other reasons; and
- Underprepared student: those who lack the required requisite background for a subject.

What follows are the results of visiting mathematics department websites and other internet sites to find ways to help all students to develop study plans and the resources to be independent learners.

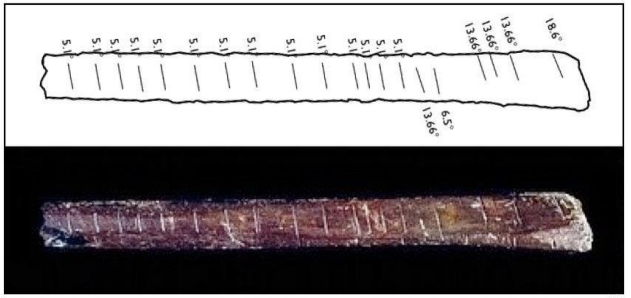
Bridging the Gaps on the History of Undergraduate Mathematics

Every culture has contributed to the history of mathematics. Every student has an opportunity to know of the contributions to mathematics made by various cultures. The internet opens the doors for all cultures to record their contributions to the history of mathematics. The following are examples of the many sites that provide a relatively comprehensive presentation of the contributions to mathematics by many cultures.



Timeline of Undergraduate Mathematics

What follows is an abbreviated timeline derived from Wikipedia. Contributions by different cultures have also been included. https://en.wikipedia.org/wiki/Timeline_of_mathematics



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-44,200 The Lebombo bone is a bone tool made of a baboon fibula with incised markings discovered in the Lebombo Mountains near **Swaziland**. The bone is between 44,200 and 43,000 years old, according to radiocarbon datings. The Lebombo bone's 29 notches suggest that it may have been used as a lunar phase counter. It is the oldest known mathematical artifact.

-20,000 The Ishango bone is a bone tool. It is the fibula of a baboon, with a sharp piece of quartz affixed to one end, perhaps for engraving or writing. It was first thought to be a tally stick, as it has a series of tally marks carved in three columns running the length of the tool, but some scientists have suggested that the groupings of notches indicate a mathematical understanding that goes beyond counting. The bone was discovered in the African area of Ishango, on the border between modern-day Uganda and Congo.

https://afrolegends.com/2013/08/29/the-ishango-bone-craddle-of-mathematics/

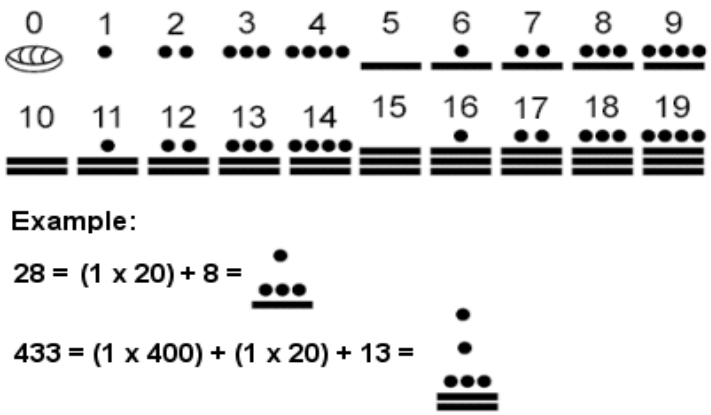
-3400 **Mesopotamia (present-day Iraq),** the Sumerians invent the first known numeral system, and a system of weights and measures.

- 3100 **Egypt** developed the earliest known decimal system that allows indefinite counting by way of introducing new symbols.

-2400 **Egypt** developed a precise astronomical calendar, used even in the Middle Ages for its mathematical regularity.

-2000 The **ancient Chinese numbering system** was a decimal place value system, very similar to the one we use today - it was the first such number system, adopted by the Chinese over a thousand years before it was adopted in the West - and it made even quite complex calculations very quick and easy.

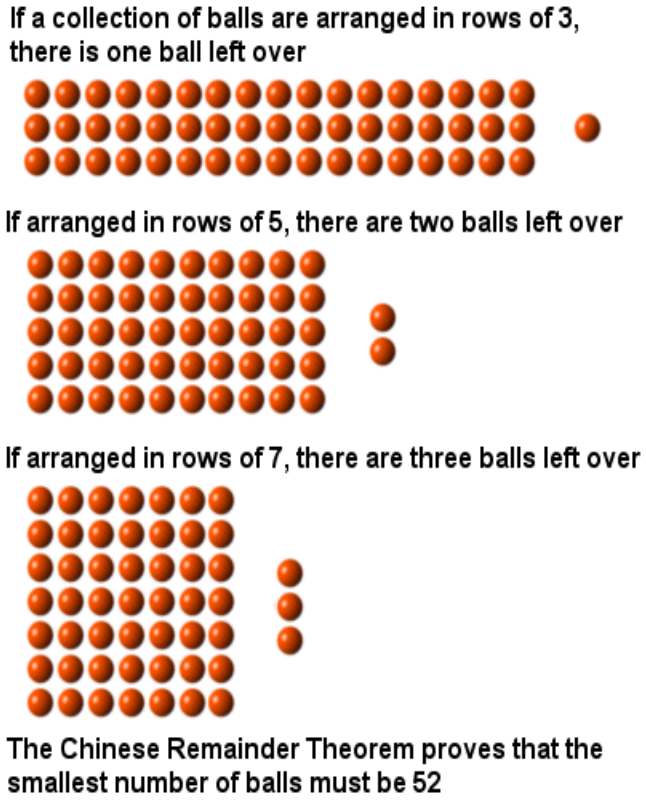
-2000 **Mayan Mathematics:** The importance of astronomy and calendar calculations in the Mayan society required mathematics, and the Maya constructed quite early a very sophisticated number system, possibly more advanced than any other in the world at the time.



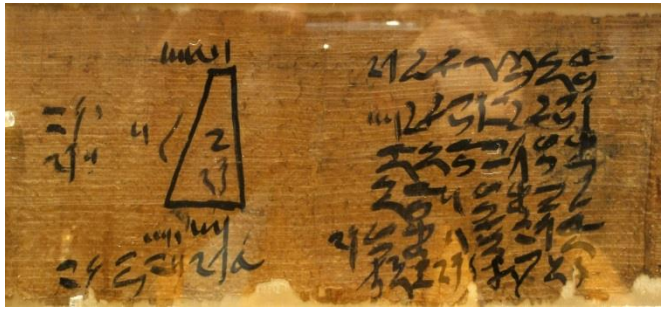
-2000 A Chinese textbook translated as “the Nine Chapters on the Mathematical Art” covered hundreds of problems in areas such as trade and engineering. It was important as a guide to how to solve equations - the deduction of an unknown number from other known information - using a sophisticated matrix-based method which did not appear in the West until Carl Friedrich Gauss re-discovered it at the beginning of the 18 th Century (and which is now known as Gaussian elimination).

Among the greatest mathematicians of Ancient China was Liu Hui, who by an approximation using a regular polygon with 192 sides, formulated an algorithm which calculated the value of π as 3.14159 as well as developing a very early forms of both integral and differential calculus.

The Chinese also pursued more abstract mathematical problems including the Chinese Remainder Theorem.



-1,890 The **Moscow Mathematical Papyrus** is from ancient Egypt. The text was estimated to be written down in the Twelfth dynasty of Egypt. The papyrus is well known for some of its geometry problems. Problems 10 and 14 compute a surface area and the volume of a frustrum respectively.



-1650 The first part of the **Rhind Papyrus** from Edypt consists of reference tables and a collection of 21 arithmetic and 20 algebraic problems. The second part of the Rhind papyrus, being problems 41–59 and 60, consists of geometry problems.

-1,200 **Chou-pei Suan Ching** is one of the oldest Chinese mathematical classics and contains one of the first recorded proofs of the Pythagorean Theorem

-1000 Mantras (in Hinduism and Buddhism) from the early Vedic period invoke powers of ten from a hundred all the way up to a trillion, and provide evidence of the use of arithmetic operations such as addition, subtraction, multiplication, fractions, squares, cubes and roots.

-1000 As early as the 3rd or 2nd Century BCE, Jain mathematicians recognized five different types of infinities: infinite in one direction, in two directions, in area, infinite everywhere and perpetually infinite, demonstrate an awareness of indeterminate and infinite numbers, with numbers deemed to be of three types: countable, uncountable and infinite. Brahmagupta established the basic mathematical rules for dealing with zero: 1 + 0 = 1; 1 - 0 = 1; and 1 x 0 = 0. He also established rules for dealing with negative numbers, and pointed out that quadratic equations could in theory have two possible solutions, one of which could be negative.

-540 Pythagoras: The society of Pythagoreans are credited with the proof of the existence of irrational numbers and many other mathematical proofs.

-367 The Musaeum was an institution founded, at Alexandria, Egypt. It was the prototype of the modern day university.

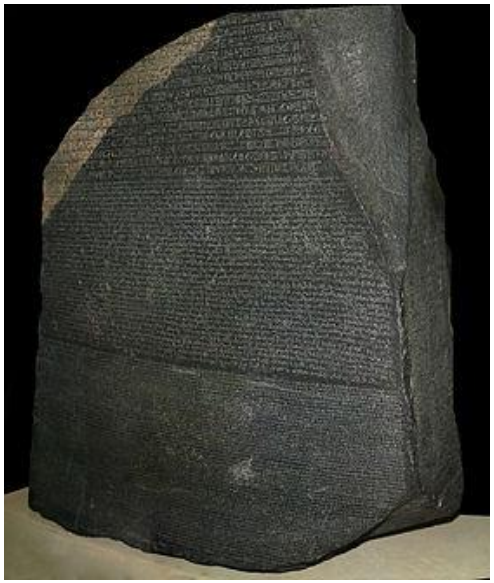
-332 The Library of Alexandria was one of the largest and most significant libraries of the ancient world. The Library was part of the larger research institution, the Musaeum. It is estimates that over 700,000 scrolls at its height.

https://www.youtube.com/watch?v=JyRslVAS6CE

-300 **Euclid's Elements** consisted of 13 books that covers geometry and number theory. It is one of the only books that survived in entirety from the burning of the Library of Alexandria, serving as the main textbook for teaching mathematics (especially geometry) from the time of its publication until the early 20th century.

-230 **Sieve of Eratosthenes** is an algorithm for finding all prime numbers up to any given limit.

-196 The **Rosetta Stone** contains Ancient Egyptian hieroglyphs, Demotic script, and Greek script. The **Rosetta Stone** was discovered in 1799 and which is inscribed with three versions of a decree issued at Memphis, Egypt on behalf of King Ptolemy V. The Rosetta Stone became the key to deciphering Egyptian hieroglyphs, thereby opening a window into ancient Egyptian history.



-150 A method of Gaussian elimination appears in the Chinese text: ***The Nine Chapters on the Mathematical Art***.

-48 Burning of the Library of Alexandria

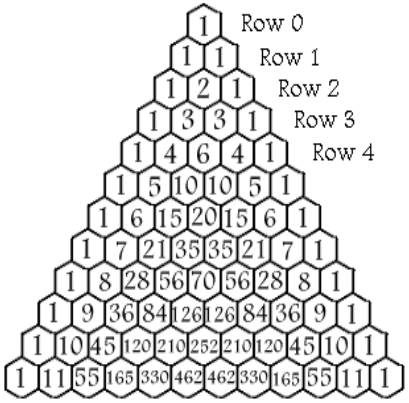
820 **Al-Khwarizmi**– Persian (Islamic Republic of Iran) mathematician, is credited as the Father of algebra, writes the *Al-Jabr*, later transliterated as *Algebra*, which introduces systematic algebraic techniques for solving linear and quadratic equations.

953 **Muhammad Al-Karaji** was the first to use the method of proof by **mathematical induction** to prove his results, by proving that the first statement in an infinite sequence of statements is true, and then proving that, if any one statement in the sequence is true, then so is the next one. Among other things, Al-Karaji used **mathematical induction** to prove the **binomial theorem**.

The Binomial Theorem can be stated as:

$$(a+b)^n = a^n + na^{n-1}b^1 + \frac{n(n-1)}{2}a^{n-2}b^2 + \ldots\ldots + b^n$$

The co-efficients generated by expanding binomials of the form $(a+b)^n$ can be shown in the form of a symmetrical triangle:



1571 **Johannes Kepler** discovered the three laws of **planetary motion**.

1623 **Blaise Pascal** was an important mathematician who helped to create **projective geometry and later probability theo**ry.

1641 **Renee Descartes**: In his book, The Method of Discourse, he **developed analytic geometry**, **which uses algebra to describe geometry**. Descartes "invented the convention of representing unknowns in equations by *x*, *y*, and *z*, and knowns by *a*, *b*, and *c*".

1642 **Sir Isaac Newton developed Calculus independently of Leibniz**.

1646 **Gottfried Wilhelm (von) Leibniz developed differential and integral calculus independently of Newton**

1736 Leonard Euler: Topology, as a well-defined mathematical discipline, originates in the early part of the twentieth century, but some isolated results can be traced back several centuries. Among these are certain questions in geometry investigated by Leonhard Euler in his paper Seven Bridges of Königsberg is regarded as one of the first practical applications of topology

Bridging the Gaps in Education

Some educational policies have allowed for the creation of learning disparities by introducing, for example, slow learner's classes versus gifted programs or multiple tracks such as labor force curricula versus college preparation curricula. Do such policies allow for equal opportunity in education or equal opportunities for careers for all students?

The open-door academic policy has enabled a step into higher education such as a bachelor's degree, to those who had restricted access to these opportunities due to social or economic factors. The policy has also created a sufficient amount of well-trained students to fulfill the demand for educated employees for the STEM and business industries.

However, despite its benefits the open-door academic policy has faced its criticism. The graduation rates of colleges are closely tied to their admissions policies. Six years after beginning a four-year program, the rate varies from 89% at colleges that accept less than one-quarter of applicants to 36% at those with an open admissions policy.

Fortunately, the technology that is available to teachers and students today can make a significant impact on student achievement. Educators are able to embrace a blended approach to teaching that combines technology with traditional instruction. With a blended approach, students can learn online while also receiving support and instruction from their teachers through lectures.

Educators have the ability to become culturally proficient by understanding of the various cultural backgrounds that students bring to the classroom. Teachers should aim to embrace the cultural differences within the classroom and develop teaching tools and communication skills that enable students from all backgrounds to succeed.

Sandra Kingan and Jeff Suzuki of Brooklyn College, with the support of a NSF grant, created a course to develop a curriculum that builds a community of students and faculty who are knowledgeable about mathematics and social advocacy, in the sense that they are able to act as advocates for societal issues they care about. The website contains of many lessons and problems in college algebra and statistics on social issues.

https://sites.google.com/site/jeffsuzukiproject/mathematics-and-social-advocacy

Through the use of technology, students see content in many forms through videos, hyperlinks to definitions and more. Teachers can spend more time reviewing the performance of each student and creating customized plans for intervention. Students become active participants in their own learning. Technology customizes to provide gifted students with more challenging problems and extension activities, while multiple representations of a concept are provided to uder-prepared students. Technology that adjusts the path of learning for each student and provides multiple learning modes for students to practice skills has been shown to dramatically boost achievement. (Schacter, 2001; Sivin- Kachala, 1998; Wenglinsky, 1998).

Bridging the Gaps between High School and College Through Summer Bridge Programs Saturday Academies and Math Competitions

Summer Bridge Programs are able to offer enrichment courses in mathematics, science, written and oral communication, career research, personal and professional development in activities that support college planning for students.

The City University of New York, College Access: Research and Action Bridge Program

And

Bridge to Enter Advanced Mathematics (BEAM). BEAM serves students from low-income neighborhoods in Los Angeles and New York City.

https://caranyc.org/college-bridge/

https://www.beammath.org/

BEAM

Bridge to Enter Advanced Mathematics

Summer Positions Available!

This summer, *change the lives* of underserved students with exceptional potential in mathematics.

Bridge to Enter Advanced Mathematics is a free program for students from low-income and historically marginalized communities who show exceptional potential in mathematics.

"Teaching at [BEAM] was a great joy, and I highly recommend it as an outreach initiative to get involved in!"
- Dr. Mohamed Omar, Associate Professor, Harvey Mudd College

For Summer 2019, we are hiring:

...college professors and classroom teachers as faculty. Design your own courses on favorite math topics. Teach to small classes of motivated middle schoolers.

...graduate students as junior faculty who design and teach courses with structured support and mentorship.

...college students as student life counselors and teaching assistants.

Dr. Karen Taylor works with a student in her Number Theory class.

For more information and how to apply:

beammath.org/jobs

Proposal:
High School Bridge Program in Mathematical Logic and Set Theory

A High School Program to offer mathematical theory in an informal setting.

Students would be introduced to the theory of mathematics before college through games, lectures competitions, presentations and puzzles in:

- Logic and Set Theory
- Mathematical Induction
- Geometry and trigonometry
- Mathematical proofs
- Computer languages

Examples of High School Mathematics Competitions and Leagues

- **List of United States high school mathematics competitions**
- https://artofproblemsolving.com/wiki/index.php/List_of_United_States_high_school_mathematics_competitions

- **American Regions Mathematics League**
- https://artofproblemsolving.com/wiki/index.php/American_Regions_Mathematics_League

- The **iTest** (formerly the American High School Internet Mathematics Competition)
- https://artofproblemsolving.com/wiki/index.php/ITest

- Association of Computational and Mathematical Modeling https://aocmm.org/

- MathWorks Math Modeling (M3) Challenge Moody's Mega Math Challenge

- Art of Problem Solving Art of Problem Solving

Bridging the Gaps in Undergraduate Mathematics Courses

Introduce texts and topics that are culturally relevant

Technology is a great way to build in personalized learning as students can begin to pace themselves, find new challenges independently, and work collaboratively with resources that will lead to college and career success. Personalized learning works best when you have a model in your classroom that allows students to focus on different learning needs.

Suggestions for Rewarding and Motivating Success

At the beginning of the semester, student can be provided with suggestions and strategies for studying Mathematics. In order to develop good reading and writing habits for mathematics courses, students can be rewarded for taking good notes and for home work.

Regular comprehensive open notes quizzes that are challenging yet allow for extra credit points, can serve as an incentive for students to not give up or quit. Immediate feed-back with solutions to quizzes with sufficient detail for all backgrounds, helps students who are under-prepared to be able to improve their chances of passing while being evaluated on challenging problems that reflect a “national standard”.

Comprehensive Tests

Chapter tests can be designed to cover all of the essential topics and problems .
Test scores may be scaled to allow for students to earn extra credit points. This allow for a student's average score to improve based with his or her progress. The maximum amount of available time is allowed for tests.

Test Corrections

Students are rewarded for correcting their tests. In helping students to develop good writing habits they can be asked to:

a. Write test corrections on paper separate from the exam.
b. State each question for which one receives partial or no credit.
c. Provide a complete solution with all steps.
d. Circle the final answer.
e. This becomes a study guide for the final

Showcase Good Student Performances

Student assignments and tests can be displayed on the courses' website to provide examples of good work and the standard of work that is expected of the students.

Bridging the Gaps between Undergraduate School and Graduate School

Motivations and Applications to Real World Problems

- Many graduate schools place past Ph.D qualifying exams online.
- Articles in mathematics journals that apply concepts taught in undergraduate courses can be found online.
- GRE Mathematics Test Practice Books and solutions can be found online.

Summer REU's help prepare students for graduate studies by participating in research projects.

SAMMS (Sampling Advanced Mathematics for Minority Students)

SAMMS is a summer school for undergraduate students, primarily from traditionally underrepresented students. SAMMS is organized by The Ohio State University and the University of Puerto Rico, Mayagüez, and takes place at the Ohio State mathematics department. The program consists of a four week mini-course curriculum with additional graduate school preparation activities. For more information.

The National Alliance for Doctoral Studies in the Mathematical Sciences
Our goal is to be sure that every underrepresented or underserved American student with the talent and the ambition has the opportunity to earn a doctoral degree in a mathematical science.

Mathematics Advanced Study Semesters
The Mathematics Advanced Study Semesters (MASS) program is held during the Fall semester of each year at Pennsylvania State University. For most of its participants, the MASS program serves as a spring board to graduate schools in mathematics.

The program consists of three courses chosen from major areas in Algebra, Analysis, and Geometry respectively, specially designed and offered exclusively to MASS participants, and a weekly working seminar.

Enhancing Diversity in Graduate Education (EDGE) for Women
The EDGE Program is administered by the Sylvia Bozeman and Rhonda Hughes EDGE Foundation. This four week programs offers courses in abstact algebra and real analysi
Along with the summer session, EDGE supports an annual conference, travel for research collaborations, travel to present research and other open-ended mentoring activities.

The MSRI Undergraduate Program
(MSRI-UP) is a comprehensive summer program designed for undergraduate students who have completed two years of university-level mathematics courses and would like to conduct research in the mathematical sciences.

The NSF Bridge-to-Math-Doctorate at the University of Texas Arlington
The programs allows qualified undergraduate seniors to spend 12 months at UT Arlington while being a full-time student at their own home institution and provides a stipend of $10,000 for the year. The participating seniors are mentored and involved in coursework during the two academic semesters and research in the summer with the goal of preparation for doctoral studies.

Example of the use of Educational Technology to complement classroom lectures.

References

World Wide Web

Education Technologies

Thank you

The Florida A&M University Department of Mathematics

The National Association of Mathematicians, Inc.

The Department of Economics and
The Department of Mathematics and Statistics at Washington University

The Math Alliance 2019 Field of Dreams Conference