

What is...Coding Theory?

Field of Dreams 2017
November 4th, 2017

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Redundancy



redundancy



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re·dun·dan·cy

/rəˈdʌndənsē/

noun

the state of being not or no longer needed or useful.

"the redundancy of 19th-century heavy plant machinery"

- the use of words or data that could be omitted without loss of meaning or function; repetition or superfluity of information.

synonyms: [superfluity](#), unnecessaryness, [excess](#)

"redundancy in language"

- **ENGINEERING**

the inclusion of extra components that are not strictly necessary to functioning, in case of failure in other components.

"a high degree of redundancy is built into the machinery installation"

Dre ms

Back to “redundancy”

Using 26 letters, the number of possible “words” that are ten letters long is

$$26^{10} = 141,167,095,653,376$$

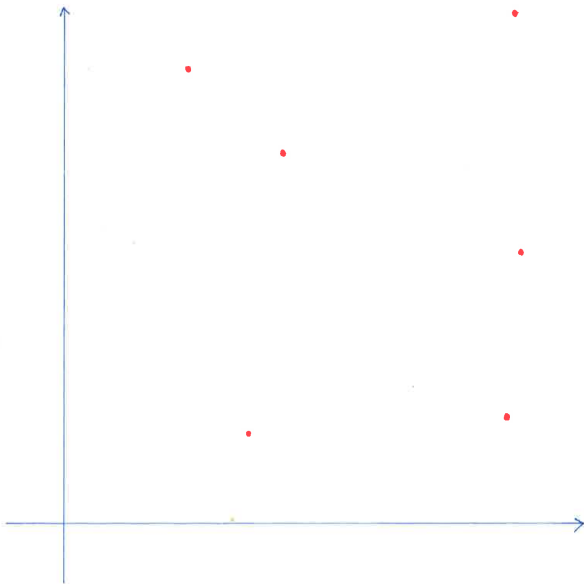
e.g. ygfetodskg

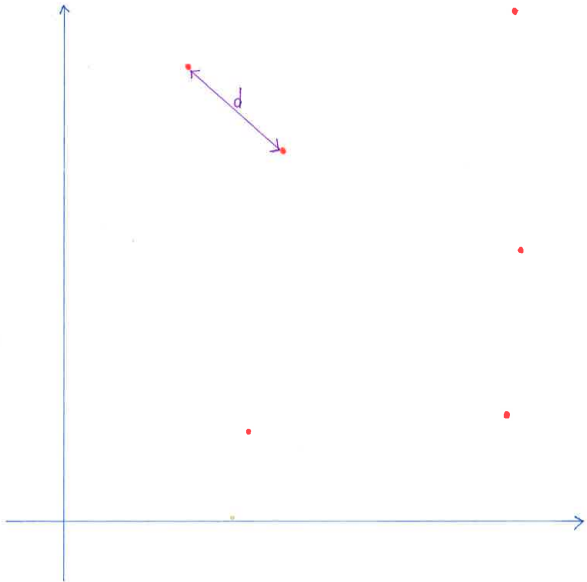
Back to “redundancy”

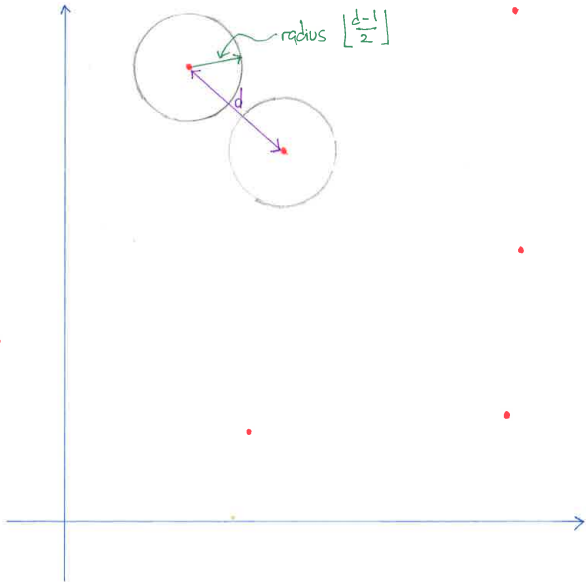
BUT: there are only

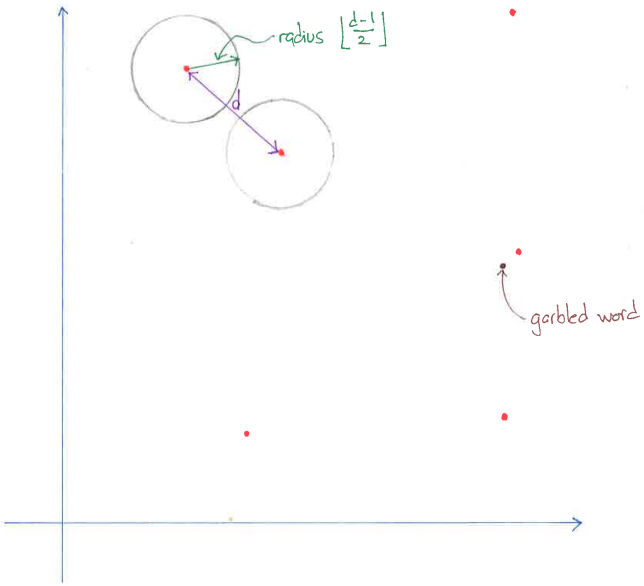
35,529

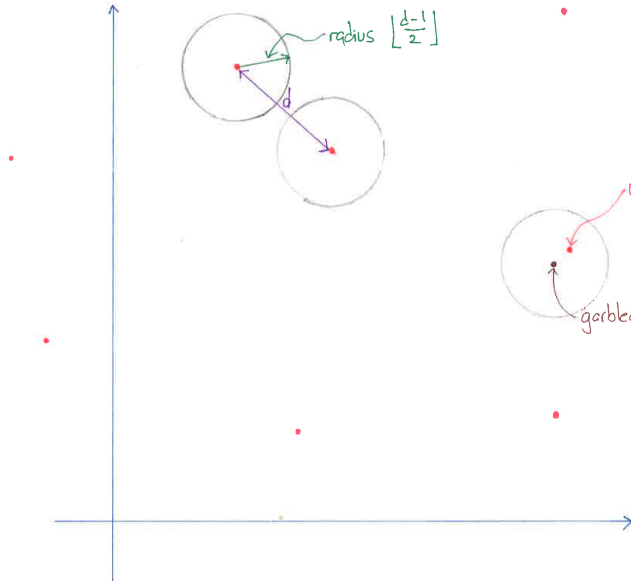
ten-letter words in the English language.











unique honest word
in neighborhood of
garbled word.

garbled word

Coding Theory makes the metaphor precise

Build redundancy into data to:

1. Detect errors in data transmission.
2. Correct errors in data transmission.
3. Repair erasures in data transmission/storage.

A code on the alphabet $\{0,1\}$

Say we use words of length $n = 6$.

Code words:

$(0,0,0,0,0,0)$, $(0,0,0,1,1,1)$,

$(1,1,1,0,0,0)$, $(1,1,1,1,1,1)$.

Information rate: $2/6 = 1/3$.

Dimension of code is $k = (1/3) \times 6 = 2$.

Data corruption

Suppose you receive the following message:

$$(0, 1, 0, 0, 0, 0)$$

Which of the codewords

$$(0, 0, 0, 0, 0, 0), (0, 0, 0, 1, 1, 1),$$
$$(1, 1, 1, 0, 0, 0), (1, 1, 1, 1, 1, 1).$$

do you think you were *meant* to receive?

Minimum distance

A (linear) code with **minimum distance** d can

1. detect up to $d - 1$ many errors.
2. correct up to

$$\left\lfloor \frac{d-1}{2} \right\rfloor$$

many errors.

Data erasure

Suppose you receive the following message:

$$(1, 1, \text{ }, 0, 0, 0)$$

Which of the codewords

$$(0, 0, 0, 0, 0, 0), (0, 0, 0, 1, 1, 1),$$
$$(1, 1, 1, 0, 0, 0), (1, 1, 1, 1, 1, 1).$$

do you think you were *meant* to receive?

Locality

A code has **locality** r if any of its symbols can be reconstructed from knowledge of r other symbols.

Fundamental Constraint

The parameters $[n, k, d, r]$ of a code are subject to:

$$n - k - \left\lceil \frac{k}{r} \right\rceil + 2 \geq d.$$

Recent work (2017)

We have constructed on an alphabet of 7 letters, with parameters

$$[n, k, d, r] = [48, 31, 3, 2].$$

Note that

$$48 - 31 - \left\lceil \frac{31}{2} \right\rceil + 2 = 3.$$

Who's "we"?

Alexander Barg, University of Maryland

Kathryn Haymaker, Villanova University

Everett Howe, Center for Communications Research

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Applications

1. Distributed storage systems.
2. Cloud storage.

What do I read now?

Judy Walker, *Codes and Curves*.